Solving the BFKL Equation with Running Coupling

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Abstract

We describe a formalism for solving the BFKL equation with a coupling that runs for momenta above a certain infrared cutoff. By suitably choosing matching conditions proper account is taken of the fact that the BFKL diffusion implies that the solution in the infrared (fixed coupling) regime depends upon the solution in the ultraviolet (running coupling) regime and vice versa. Expanding the BFKL kernel to a given order in the ratio of the transverse momenta allows arbitrary accuracy to be achieved.

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Diffractive processes are described, at least within the framework of perturbative QCD, by the BFKL equation [1]. This equation sums the leading and next-to-leading $\ln s$ [2] terms for the amplitude involving the exchange in the $t-$channel with the quantum numbers of the vacuum.

In the case of fixed coupling, the energy dependence of these diffractive processes is determined by the emergence of a branch cut in the Mellin transform plane ($\omega-$plane), with a branch point at $\omega > 0$. Lipatov [3] has demonstrated that accounting for the running of the strong coupling together with some (non-perturbative) information about the infrared behaviour of QCD leads to a separation of the $\omega-$plane singularity structure into a series of isolated poles. More recently, Thorne [4] has shown that if one uses the solution to the BFKL equation with running coupling, the factor in the amplitude which is totally calculable in perturbative QCD has an essential singularity at $\omega = 0$, such that there is no power-like energy dependence. This is in accord with the recent work of Ciafaloni, Colferai and Salam, who have shown that the leading ω-plane singularity is of non-perturbative origin [5].

The BFKL equation is a diffusion equation and so there will always be some contribution from the infrared region. One way of treating this region is to assume that the strong coupling ceases to run below a certain scale. In this case, we expect that for sufficiently low values of the transverse momenta of the exchanged gluons, the $s-$behaviour will be dominated by the fixed coupling solution, whereas for large enough transverse momenta the solution of [4] will dominate. We should like to understand the smooth extrapolation between these two extremes.

In this letter we describe a formalism for solving the BFKL equation with strong coupling running above some infrared scale, in which we can see how the transition from the fixed coupling to the running coupling solution arises.

Formalism for Solving the BFKL Equation with Running Coupling

To begin with we consider only the leading order BFKL kernel and describe later how the formalism may be extended in order to include the substantial part of the higher order corrections.

Since the BFKL equation is a diffusion equation there is always diffusion into the infrared regime, where renormalization group improved perturbation theory breaks down and one needs supplementary information about the behaviour of QCD beyond perturbation theory. For our purposes we simulate this infrared behaviour by assum-

ing that the coupling freezes below a critical value of $t \equiv \ln(k^2/\Lambda_{QCD}^2)$, $t = t_0$. We also make the assumption that $\alpha_s(t_0)$ is sufficiently small that perturbative results in the infrared regime maintain some level of credibility.

The full BFKL equation is solved in terms of a complete set of eigenfunctions $f_\omega(t)$ of the kernel with eigenvalue ω. For any ω there will be some diffusion into the infrared regime. We therefore write $f_\omega(t)$ as

$$f_\omega(t) = f_\omega^<(t)\,\theta(t_0 - t) + f_\omega^>(t)\,\theta(t - t_0) \tag{1}$$

and the kernel as

$$
\begin{aligned}
K(t,t') &= \left[\frac{1}{b\,t_0}\mathcal{K}^<(t,t')\,\theta(t_0 - t') + \frac{1}{b\,t}\mathcal{K}^<(t,t')\,\theta(t' - t_0)\right]\theta(t - t') \;+ \\
&\quad \left[\frac{1}{b\,t_0}\mathcal{K}^>(t,t')\,\theta(t_0 - t') + \frac{1}{b\,t'}\mathcal{K}^>(t,t')\,\theta(t' - t_0)\right]\theta(t' - t)
\end{aligned}
\tag{2}
$$

where $b = \beta_0/12$, $\beta_0 = 11 - 2n_f/3$, being the first coefficient of the $\beta-$function for the running of the QCD coupling. We define $\mathcal{K}^<(t,t')$ and $\mathcal{K}^>(t,t')$ as the limits of sums, i.e.

$$
\begin{aligned}
\mathcal{K}^<(t,t') &= \lim_{N\to\infty} \mathcal{K}_N^<(t,t') \\
\mathcal{K}^>(t,t') &= \lim_{N\to\infty} \mathcal{K}_N^>(t,t')
\end{aligned}
$$

where

$$\mathcal{K}_N^<(t,t') = \sum_{r=0}^N \left\{e^{(r+1/2)(t'-t)} - \frac{1}{r+1}\delta(t - t')\right\} \tag{3}$$

$$\mathcal{K}_N^>(t,t') = \sum_{r=0}^N \left\{e^{(r+1/2)(t-t')} - \frac{1}{r+1}\delta(t - t')\right\}. \tag{4}$$

The functions $e^{\gamma t}$ are eigenfunctions of these kernels with eigenvalues $\chi_N^<(\gamma)$ and $\chi_N^>(\gamma)$ respectively, i.e.

$$
\begin{aligned}
\int_{-\infty}^t \mathcal{K}_N^<(t,t')e^{\gamma t'}dt' &= \chi_N^<(\gamma)e^{\gamma t} \\
\int_t^\infty \mathcal{K}_N^>(t,t')e^{\gamma t'}dt' &= \chi_N^>(\gamma)e^{\gamma t}
\end{aligned}
\tag{5}
$$

where

$$\chi_N^<(\gamma) = \sum_{r=0}^N \left\{\frac{1}{r+1/2+\gamma} - \frac{1}{r+1}\right\} \tag{6}$$

$$\chi_N^>(\gamma) = \sum_{r=0}^N \left\{\frac{1}{r+1/2-\gamma} - \frac{1}{r+1}\right\}. \tag{7}$$

Note that $\chi_\infty^<(\gamma) = \gamma_E - \psi(1/2 + \gamma)$ and $\chi_\infty^>(\gamma) = \gamma_E - \psi(1/2 - \gamma)$, and we have the leading order BFKL kernel.

In this paper we truncate the kernel after N terms. Note that the case $N = 0$ reduces to the collinear model of [5]. For this truncated kernel the spectrum of eigenvalues is

$$-\sum_{r=0}^{N} \frac{2}{r+1} \; \leq \; b \, t_0 \, \omega \; \leq \; \sum_{r=0}^{N} \frac{2}{(2r+1)(r+1)}. \tag{8}$$

The infrared part of the eigenfunction $f_\omega^<(t)$ obeys the integral equation

$$b \, t_0 \, \omega \, f_\omega^<(t) \;=\; \int_{-\infty}^{t} \mathcal{K}_N^<(t,t') f_\omega^<(t') \, dt' \;+\; \int_{t}^{t_0} \mathcal{K}_N^>(t,t') f_\omega^<(t') \, dt'$$
$$+\; \int_{t_0}^{\infty} \mathcal{K}_N^>(t,t') \frac{t_0}{t'} f_\omega^>(t') dt' \tag{9}$$

whereas the ultraviolet part of the solution $f_\omega^>(t)$, obeys the integral equation

$$b \, t \omega \, f_\omega^>(t) \;=\; \int_{t_0}^{t} \mathcal{K}_N^<(t,t') f_\omega^>(t') \, dt' \;+\; \int_{t}^{\infty} \mathcal{K}_N^>(t,t') \frac{t}{t'} f_\omega^>(t') \, dt'$$
$$+\; \int_{-\infty}^{t_0} \mathcal{K}_N^<(t,t') f_\omega^<(t') \, dt'. \tag{10}$$

We note that these equations are both inhomogeneous, reflecting the fact that even for $t < t_0$, where the coupling is fixed, the BFKL equation involves diffusion into the running coupling regime and vice versa.

We can convert these two equations into homogeneous integro-differential equations by operating on both sides of eq.(9) with the operator

$$\mathcal{O}_<^N(t) \;=\; \frac{d}{dt}\left(\frac{d}{dt} - 1\right) \cdots \left(\frac{d}{dt} - N\right) e^{-t/2}$$

and operating on both sides of eq.(10) with the operator

$$\mathcal{O}_>^N(t) \;=\; \frac{d}{dt}\left(\frac{d}{dt} + 1\right) \cdots \left(\frac{d}{dt} + N\right) e^{t/2}.$$

These equations are $(N+1)^{th}$ order in derivatives and so each one of these has solutions which contain $N+1$ arbitrary constants of integration. However, these constants are fixed by the requirement that the original integral equations (9,10) must be obeyed, together with the requirements the eigenfunctions should be normalizable, which means that $f_\omega^<(t)$ must be square integrable as $t \to -\infty$ and $f_\omega^>(t)$ must be square integrable as $t \to \infty$.

The general solution to the integro-differential equation for $f_\omega^<(t)$ is

$$f_\omega^<(t) \;=\; \sum_{j=1}^{N+2} a_j e^{\gamma_j^0 t}, \tag{11}$$

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where γ_j^0 are the solutions to

$$b\,\omega\,t_0 \;=\; \chi_N^<(\gamma) + \chi_N^>(\gamma) \;\equiv\; \chi_N(\gamma) \tag{12}$$

with $\Re e(\gamma) \geq 0$ to ensure square integrability as $t \to -\infty$. In the allowed range of eigenvalues, eq.(8), there will be $N+2$ such solutions. For example, in the case $N = 0$ the solutions are

$$\gamma_j^0 \;=\; \pm i\,\sqrt{\left(\lambda_0 - \frac{1}{4}\right)} \tag{13}$$

where

$$\lambda_0 = \frac{1}{b\,\omega\,t_0 + 2}.$$

For the allowed range, $\lambda_0 > 1/4$, and so these values are purely imaginary. In the case $N = 1$ we have

$$\gamma_j^0 \;=\; \pm\sqrt{\left(\frac{10}{8} - 2\lambda_1 \pm \sqrt{1 - 2\lambda_1 + 4\lambda_1^2}\right)} \tag{14}$$

where

$$\lambda_1 = \frac{1}{b\,\omega\,t_0 + 3}.$$

For the allowed range, $\lambda_1 > 3/16$ and we see that two of the solutions are purely imaginary, $\pm i\nu_1$, and two purely real $\pm\beta_1$. Thus the general solution is

$$f_\omega^<(t) \;=\; a_1 e^{i\nu_1 t} \,+\, a_2 e^{-i\nu_1 t} \,+\, a_3 e^{\beta_1 t}. \tag{15}$$

For the integro-differential equation for $f_\omega^>(t)$, it is convenient to work in terms of the Laplace transform $\tilde{f}_\omega(\gamma)$ defined by

$$\frac{f_\omega^>(t)}{t} \;=\; \frac{1}{2\pi i} \int d\gamma\, \tilde{f}_\omega(\gamma) e^{\gamma t}, \tag{16}$$

in which the integral over γ is performed over some suitable contour, described below. The function $\tilde{f}_\omega(\gamma)$ obeys the differential equation

$$\omega\, b\, \tilde{f}_\omega''(\gamma) \,+\, \chi_N(\gamma)\tilde{f}_\omega'(\gamma) + \chi_N^{>'}\tilde{f}_\omega(\gamma) \;=\; 0, \tag{17}$$

where the prime indicates differentiation with respect to γ. This equation can be solved in WKB approximation yielding the result

$$\tilde{f}_\omega(\gamma) \;=\; \frac{1}{(V(\gamma,\omega))^{1/4}}\, \exp\left\{-\frac{1}{2\omega b}\int^\gamma \left(\chi_N(\gamma') + \sqrt{V(\gamma',\omega)}\right) d\gamma'\right\}, \tag{18}$$

where

$$V(\gamma,\omega) \;=\; \chi_N^2(\gamma) - 4\,\omega\, b\, \chi_N^{>'}(\gamma).$$

Note that if we set the term $\chi_N^{>\prime}(\gamma)$ to zero we recover the expression obtained in [4] for the case of deep-inelastic scattering in which the transverse momenta are ordered and the coupling is chosen to run always with t.

The solution (18) is a valid approximation except near the points where $V(\gamma)$ vanishes. We could improve the solution by expanding $V(\gamma)$ to linear order around the zeroes and matching the solutions either side to an Airy function. However, the singularities in $\tilde{f}_\omega$ at these zeroes are integrable so that such an improvement would have negligible effect on the inverse Laplace transform.

For $t < t_c$ the solution is oscillatory. The value of this critical value t_c can be estimated by approximating the integral of eq.(16) by the saddle point at

$$2\,\omega\,b\,t \;=\; \chi_N(\gamma) + \sqrt{V(\gamma,\omega)}.$$

t_c is the minimum value of t for which this has a solution for purely real γ. Again, if we were to neglect $\chi_N^{>\prime}(\gamma)$, this would occur at $\gamma = 0$ and we have approximately

$$t_c \;=\; \frac{1}{b\,\omega}\chi_N(0).$$

This oscillatory part is "matched" to the fixed-coupling solution by requiring consistency with the original integral equations (9, 10). To see how this is possible, we need to examine in more detail the possible contours of integration for eq.(16).

The γ-contour

From the positions of the simple poles in $\chi_N^{<}(\gamma)$ and $\chi_N^{>}(\gamma)$, we note that $\tilde{f}_\omega(\gamma)$ has branch points on the real axis at $\gamma = r + 1/2$ and $\gamma = -1/2 - r$ for $r = 0, \cdots N$. The branch cuts can be "combed" in the direction of the positive or negative real axis in such a way as to leave a portion of the real axis between $n - 1/2$ and $n + 1/2$ for which $\tilde{f}_\omega(\gamma)$ is analytic. n runs from $-N$ to N.

For each such "combing" a contour, $\mathcal{C}_n$ can be chosen that crosses the real axis between $n - 1/2$ and $n + 1/2$. Since $t > 0$ we require that the ends of the contour turn over so that at the ends of the contour $\Re e\,\gamma \to -\infty$, thus ensuring that the inverse Laplace transform integral exists. The integral over each such contour is a valid solution to the integro-differential equation for $f_\omega^{>}(t)$ and since this equation is linear, the most general solution which is square integrable as $t \to \infty$ is the sum of the integrals over these contours with $-N \leq n \leq 0$, with arbitrary coefficients b_n,

i.e.

$$\sum_{n=-N}^{0} b_n \int_{\mathcal{C}_n} d\gamma \tilde{f}_\omega(\gamma) e^{\gamma t}, \tag{19}$$

with $\tilde{f}_\omega(\gamma)$ given by eq.(18).[†] As an example we consider the $N = 1$ case. The four "combings" are shown in Fig. 1 together with the four contours, $\mathcal{C}_n$. Only $\mathcal{C}_{-1}$ and $\mathcal{C}_0$ give a square integrable solution, so the general solution is

$$f_\omega^>(t) \;=\; t\left\{ b_{-1} \int_{-\infty}^{\infty} d\nu\, e^{-t}\, \tilde{f}_\omega(i\nu - 1)e^{i\nu t} \;+\; b_0 \int_{-\infty}^{\infty} d\nu \tilde{f}_\omega(i\nu)e^{i\nu t} \right\}. \tag{20}$$

Returning now to the original equations (9) and (10), we see that these are obeyed by the solutions (19) and (11) provided

$$\sum_{j=1}^{N+2} a_j \frac{e^{\gamma_j^0 t_0}}{(\gamma_j^0 - r - 1/2)} - t_0 \sum_{n=-N}^{0} b_n \int_{\mathcal{C}_n} d\gamma \frac{\tilde{f}_\omega(\gamma)\, e^{\gamma t_0}}{(\gamma - r - 1/2)} \;=\; 0 \quad (r = 0 \cdots N), \tag{21}$$

$$\sum_{j=1}^{N+2} a_j \frac{e^{\gamma_j^0 t_0}}{(\gamma_j^0 + r + 1/2)} + \sum_{n=-N}^{0} b_n \int_{\mathcal{C}_n} d\gamma \frac{\tilde{f}'_\omega(\gamma)\, e^{\gamma t_0}}{(\gamma + r + 1/2)} \;=\; 0 \quad (r = 0 \cdots N). \tag{22}$$

Thus we obtain $2N + 2$ relations between the $N + 2$ coefficients a_j and the $N + 1$ coefficients b_k (the overall normalization of these coefficients is determined by the normalization condition of the eigenfunctions). This then determines uniquely the required eigenfunctions.

Equations (9, 10) guarantee the continuity of the eigenfunctions at $t = t_0$, i.e. $f_\omega^<(t_0) = f_\omega^>(t_0)$, but the derivative is not continuous. This results from the fact that we have taken an expression for the coupling as a function of t, whose derivative is not continuous at $t = t_0$.

We can now see how the inverse Mellin transform, $f(s,t)$ of the solution $f_\omega(t)$ extrapolates smoothly between the power-like behaviour for $t \sim t_0$ and the softer behaviour (with at most logarithmic dependence on s) for $t \gg t_0$.

Examining the $\gamma-$ integral for one particular contour, $\mathcal{C}_n$, and for $t > t_0$, we see that this will be dominated by a saddle point $\gamma_s(\omega, t)$, which is the solution to

$$2b\,\omega\,t \;=\; \chi_N(\gamma) + \sqrt{V(\gamma, \omega)}. \tag{23}$$

This is multi-valued and the dominant saddle point will be the one closest to the integration contour. We call this saddle point $\gamma_s^n(\omega, t)$. In other words, $f_\omega^>(t)$ of

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[†]We have absorbed the factor of $2\pi i$ in the denominator in eq.(16) into the definition of the coefficients b_n.
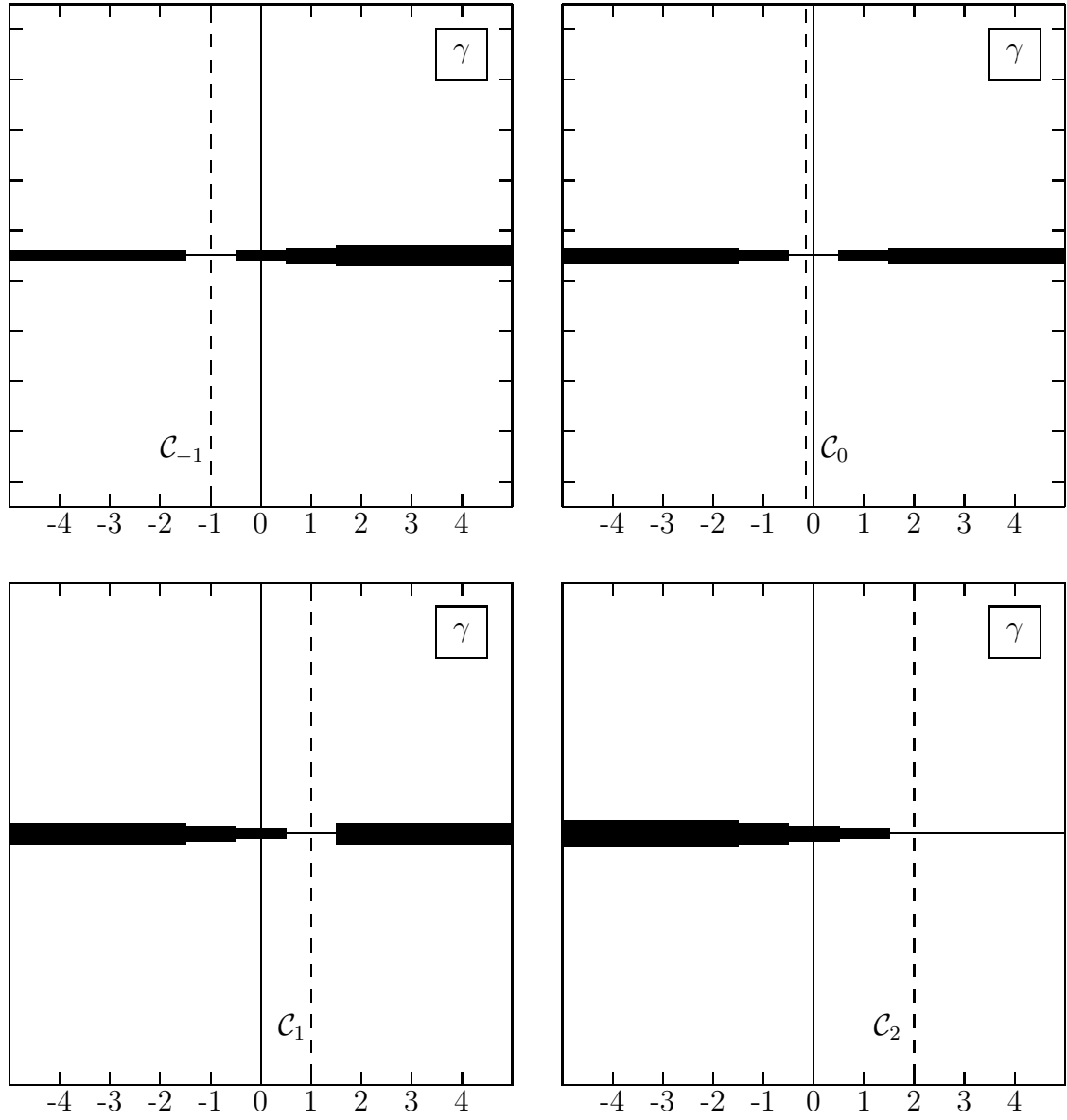
Figure 1: The four contours for the case $N = 1$ corresponding to the four different ways of "combing" the branch cuts.

eq.(16) from the contour $\mathcal{C}_n$ will contain an $\omega-$dependent prefactor, which may be written (after a suitable integration by parts)

$$\exp\left\{\int^t \gamma_s^n(\omega, t')dt'\right\}.$$

The essential singularity at $\omega = 0$ is now encoded in $\gamma_s^n(\omega, t)$ which acquires a singularity as $\omega \to 0$, i.e. the R.H.S. of (23) can only tend to zero as $|\gamma| \to \infty$.

Now examining eqs.(21, 22), we see that the contour integrals multiplying the coefficients b_n are also dominated by a similar saddle point but with t set to t_0. Thus these coefficients appear in eqs.(21, 22) with a prefactor

$$\exp\left\{\int^{t_0} \gamma_s^n(\omega, t')dt'\right\}.$$

Therefore these coefficients also possess an essential singularity at $\omega = 0$. Furthermore b_n will contain factors of the form $\exp(\gamma_0^j(\omega)t_0)$ from the L.H.S. of (21, 22).

For t close to t_0 each term in the sum of eq.(19), i.e. the product of b_n and the contribution to $f_\omega^>(t)$ from the integral over the contour $\mathcal{C}_n$ will have a dominant $\omega-$dependence of the form

$$\exp\left\{\gamma_s^n(\omega, t_0)(t - t_0) + \gamma_0^j(\omega)t_0\right\}. \tag{24}$$

Note that due to the partial cancellation, the coefficient of the term that gives rise to the essential singularity vanishes as $t \to t_0$.

For sufficiently large s, the inverse Mellin transform probes small ω, where the term in the exponent proportional to $\gamma_s^n(\omega, t_0)$ will dominate even if $(t-t_0)$ is small and the s-behaviour is dominated by the essential singularity in the Laplace transform, $\tilde{f}_\omega(\gamma)$. However, for more moderate values of s we can neglect this term when t is sufficiently close to t_0 and the $s-$dependence will be dominated by the fixed coupling power-like behaviour. As t is increased away from t_0, the boundary in s where the soft behaviour takes over from the power-like behaviour decreases and we see that as $t \to \infty$ we recover the solution of [4].

This formalism is considerably simplified if we assume that t_0 is sufficiently large that $\exp(-t_0) \ll 1$. In this case we only need consider the right-most contour consistent with square-integrability as $t \to \infty$. This is the contour $\mathcal{C}_0$. In this case, within the approximation of performing the $\gamma-$contour integral and the inverse Mellin transform by the saddle-point method, the $s-$dependence may be written as $s^{\tilde{\omega}(s,t)}$, where
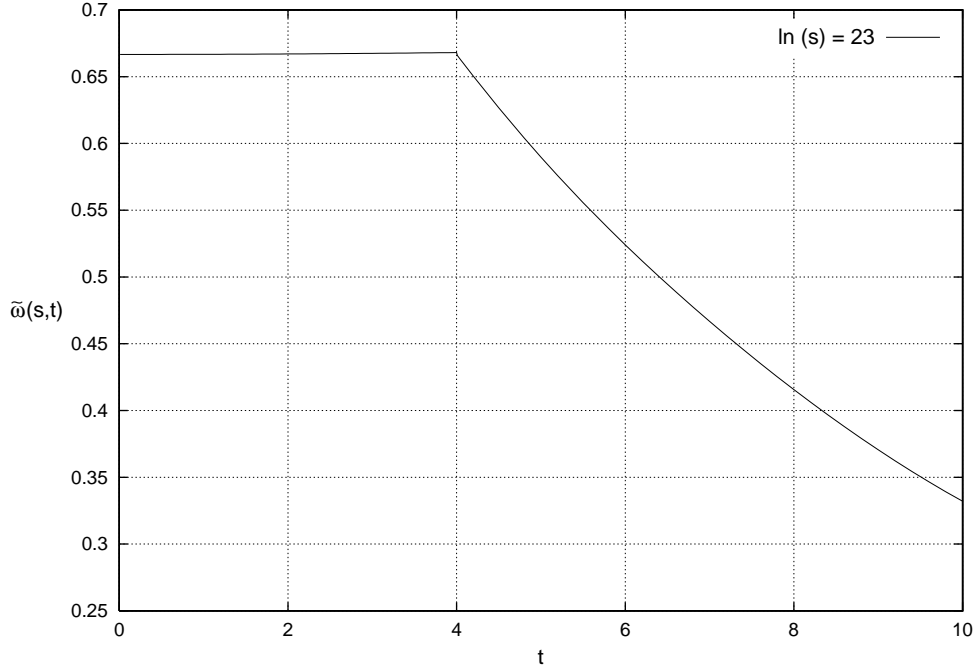
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Figure 2: $\tilde{\omega}(s,t)$ against t for $\ln s = 23$

$\tilde{\omega}(s,t)$ is the solution to

$$\ln(s) = -\frac{d}{d\omega}\left\{\theta(t-t_0)\int_{t_0}^{t}\gamma_s^0(\omega,t')dt' + \gamma_0^m(\omega)\text{Min}(t,t_0)\right\}, \qquad (25)$$

and $\gamma_0^m(\omega)$ is the solution to eq.(12) with the largest real part. In Fig. 2 we plot $\tilde{\omega}(s,t)$ against t for a large value of s ($\ln s = 23$). For simplicity we have taken the $N = 0$ case, although we expect the result to be qualitatively the same for higher values of N, and we take $t_0 = 4$. We see that it is almost constant for $t < t_0$ and diminishes as one increases t from t_0, indicating the smooth transition from hard to soft s−dependence as t is increased. The discontinuity of the slope at $t = t_0$ is a reflection of the fact that we have taken a sharp discontinuity in the slope of the running coupling at t_0 and would be smoothed out if a smoother transition were taken.

The asymptotic behaviour is only obtained for much larger values of t and in Fig. 3 we show the s dependence of $\tilde{\omega}(s,t)$ for $t = 40$ and notice that it is much closer to zero than the case where $t \sim t_0$ and decreases as $\ln s$ increases, resulting in an s−dependence for the amplitude that has no fixed power of s.

Accounting for Higher Order Corrections

The higher order BFKL kernel [2] does not lend itself readily to an expansion in

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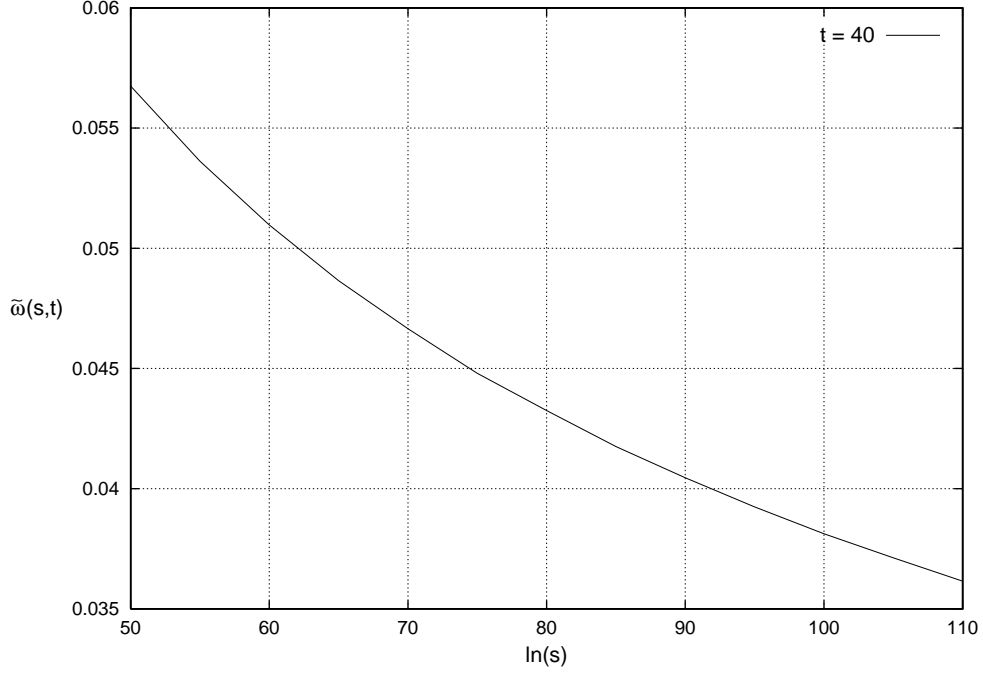
Figure 3: $\tilde{\omega}(s,t)$ against $\ln s$ for $t = 40$

powers of $e^{t-t'}$. Furthermore, it is known that these corrections are large.

However, it has been pointed out [6] that a substantial part of the higher order corrections consist of "collinear corrections" which are required to ensure correct behaviour when the formalism is applied to deep inelastic scattering. It was shown in [6] and confirmed in [7] that once these corrections are accounted for, the remaining higher order corrections are modest. The simplest way of encoding these collinear corrections and summing them is to replace γ by $(\gamma - \omega/2)$ in $\chi_N^<(\gamma)$ and by $(\gamma + \omega/2)$ in $\chi_N^>(\gamma)$ [8, 6].

This formalism is readily adapted to the procedure described here. In the expression (11) for $f_\omega^<(t)$ the quantities $\gamma_j^0(\omega)$ are taken to be the solutions to the implicit equation

$$\chi_N^<(\gamma - \omega/2) + \chi_N^>(\gamma + \omega/2) = \omega b t_0, \tag{26}$$

and the expression for the Laplace transform of $f_\omega^>(t)$ becomes

$$\tilde{f}_\omega(\gamma) = \frac{1}{(V(\gamma,\omega))^{1/4}} \exp\left\{-\frac{1}{2\omega b} \int^\gamma \left(\chi_N^<(\gamma' - \omega/2) + \chi_N^>(\gamma' + \omega/2) + \sqrt{V(\gamma',\omega)}\right) d\gamma'\right\}, \tag{27}$$

where

$$V(\gamma,\omega) = (\chi_N^<(\gamma - \omega/2) + \chi_N^>(\gamma + \omega/2))^2 - 4\omega b\, \chi_N^{>\prime}(\gamma + \omega/2). \tag{28}$$

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Provided that we restrict our analysis to the case $|\omega| < 1$, the singularity structure and consequently the analysis of the contours of integration remains unchanged from the leading order case, although the exact positions of the branch points will have moved.

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